OMR Answer Sheet No.

Question Booklet Number

7077

M.Sc. (Semester-I) Examination, 2022-23

Booklet Series

A

PHYSICS

Mathematical Physics

(To be filled in by the Candidate / निम्न विवरण परीक्षार्थी स्वयं भरें)

Roll No. (in figures) —
अनुक्रमांक (अंकों में)

Roll No. (in words)
अनुक्रमांक (शब्दों में)

Enrolment No. (in figures)

Name of College
कॉलेज का नाम

| Time : 1 : 30 Hours
| समय : 1 : 30 घण्टे

| Maximum Marks : 75
| अधिकतम अंक : 75

Signature of Invigilator
कक्ष निरीक्षक के हस्ताक्षर

Instructions to the Examinee :

1. Do not open the booklet unless you are asked to do so.
2. The booklet contains 75 questions. Examinee is required to answer any 50 questions in the OMR Answer-Sheet provided and not in the question booklet. In case Examinee attempts more than 50 Questions, *first* 50 attempted questions will be evaluated. All Questions carry equal marks.
3. Examine the Booklet and the OMR Answer-Sheet very carefully before you proceed. Faulty question booklet due to ges/questions or epancy should be

ctions on last page

परीक्षार्थियों के लिए निर्देश :

1. प्रश्न-पुस्तिका को तब तक न खोलें जब तक आपसे कहा न जाए।
2. प्रश्न-पुस्तिका मे 75 प्रश्न हैं। परीक्षार्थी को किन्हीं 50 प्रश्नों को दी गई OMR उत्तर-पत्रक पर ही हल करना है। परीक्षार्थी द्वारा 50 से अधिक प्रश्नों को हल करने की स्थिति मे, *प्रथम* 50 उत्तरों को ही मूल्यांकित किया जाएगा। सभी प्रश्नों के अंक समान हैं।
3. प्रश्नों के उत्तर अंकित करने से पूर्व प्रश्न-पुस्तिका तथा OMR उत्तर-पत्रक को सावधानीपूर्वक देख लें। दोषपूर्ण प्रश्न-पुस्तिका, जिसमें कुछ भाग छपने से छूट गये हों या प्रश्न एक से अधिक बार छप गये हों या किसी भी प्रकार की कमी हो, उसे तुरन्त बदल लें।

(शेष निर्देश अन्तिम पृष्ठ पर)

1. Two vectors are said to be collinear if :

(A) $\theta = \pi$ or $\frac{\pi}{2}$

(B) $\theta = 0$ or π .

(C) $\theta = \frac{\pi}{2}$ or $\frac{\pi}{4}$

(D) $\theta = \frac{\pi}{3}$ or $\frac{\pi}{6}$

2. Two vectors $\vec{A}$ and $\vec{B}$ are parallel if

(A) $\vec{A} \times \vec{B} = 0$.

(B) $\vec{A} \cdot \vec{B} = 0$

(C) $\vec{A} \times \vec{B} = 1$

(D) $\vec{A} \cdot \vec{B} = 1$

3. The vector Triple Product

$\vec{A} \times (\vec{B} \times \vec{C})$ is equal to

(A) $\vec{B}(\vec{A}.\vec{C}) + \vec{C}(\vec{A}.\vec{B})$.

(B) $\vec{C}(\vec{A}.\vec{B}) + \vec{B}(\vec{A}.\vec{C})$

(C) $\vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$

(D) $\vec{C}(\vec{A}.\vec{B}) - \vec{B}(\vec{A}.\vec{C})$

4. The value of $[\hat{i} \times (\hat{j} \times \hat{k}).\hat{k}]$ is

(A) $\hat{j}$

(B) $\hat{i}$

(C) 0 .

(D) $\hat{k}$

5. If the position vector of a point is $\vec{r}$, Then grad r is equal to :

(A) $\hat{r}$

(B) $r|\vec{r}|$

(C) $\frac{\vec{r}}{r}$.

(D) $\frac{\vec{r}}{r}\hat{r}$

6. If a vector $\vec{A}$ is irrotational then :

(A) Curl $\vec{A} \neq 0$

(B) div $\vec{A} = 0$

(C) Curl $\vec{A} = 0$.

(D) None of these

7. Value of Curl grad ϕ

(A) ϕ^2

(B) ϕ

(C) 1

(D) 0 .

8. What is the dimension of the vector space formed by the solution of the system of following equation?

$$2x_1+2x_2+2x_3 = 0$$
$$2x_1-2x_2+0x_3 = 0$$
$$2x_1+2x_2+2x_3 = 0$$

(A) 1

(B) 2

(C) 3

(D) 0

9. Which of the following is true?

(A) $\sqrt{(n+1)} = n\sqrt{n}$ for any real number

(B) $\sqrt{n} = n\sqrt{n+1}$ for any real number

(C) $\sqrt{(n+1)} = n\sqrt{n}$ for $n > 1$

(D) $\sqrt{n} = n\sqrt{n+1}$ for $n > 1$

10. Find the conjugate of $(6+5i)^2$

(A) (60+11i)

(B) (11−60i)

(C) (11+60i)

(D) (60−11i)

11. The conjugate of the complex number $\frac{3i+4}{2-3i}$ is

(A) $\frac{-1}{13} \quad \frac{-18}{13}i$

(B) $\frac{18}{13}i + \frac{1}{13}$

(C) $\frac{18}{13}i - \frac{1}{13}$

(D) $\frac{1}{13} - \frac{18}{13}i$

12. A Square matrix is the sum of ___ and ___

(A) Asymmetrical Matrix, Symmetrical matrix

(B) Skew symmetrical, skew symmetrical matrix

(C) Symmetrical matrix, skew symmetrical matrix

(D) None of these

13. Transpose of the matrix

$$A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$$

(A) $A^T = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -1 & 0 & -7 \end{bmatrix}$

(B) $A^T = \begin{bmatrix} 4 & 1 & -5 \\ 2 & 3 & 0 \\ -3 & -6 & -7 \end{bmatrix}$

(C) $A^T = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 7 & -6 \\ -5 & 0 & -7 \end{bmatrix}$

(D) $\begin{bmatrix} 4 & 2 & -3 \\ 7 & 3 & -6 \\ -5 & 0 & -3 \end{bmatrix}$

14. Choose the incorrect statement for the matrix

$$A = \begin{bmatrix} \omega^6 & \omega^8 & \omega^4 \\ \omega^5 & \omega^7 & \omega^9 \\ \omega^4 & \omega^5 & \omega^8 \end{bmatrix}$$

(A) A is a singular matrix

(B) A is non singular matrix

(C) A^{-1} does not exist

(D) Sum of element in each row is zero

15. The inverse of the matrix

$\begin{bmatrix} 3+2i & i \\ -i & 3-2i \end{bmatrix}$

(A) $\frac{1}{12}\begin{bmatrix} 3+2i & -i \\ i & 3-2i \end{bmatrix}$

(B) $\frac{1}{12}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

(C) $\frac{1}{14}\begin{bmatrix} 3+2i & -i \\ i & 3-2i \end{bmatrix}$

(D) $\frac{1}{14}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

16. The trace of a matrix $A=\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ is

(A) 0

(B) 4

(C) 7

(D) 5

17. The eigen values of the given matrix are given by $\begin{bmatrix} 4 & 6 \\ 2 & 8 \end{bmatrix}$

(A) (2,10)

(B) (-2,10)

(C) (3,5)

(D) (4,2)

18. The condition for which the eigen values of the matrix $A=\begin{bmatrix} 2 & 1 \\ 1 & K \end{bmatrix}$ are positive, is

(A) $K > \frac{1}{2}$

(B) $K > -2$

(C) $K > 0$

(D) $K < -\frac{1}{2}$

19. Which of the following is the characteristic equation of the matrix

$A=\begin{bmatrix} 1 & 11 & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$

(A) $A^3-16A+8A^2+10I = 0$

(B) $A^3-8A^2+10A-12I = 0$

(C) $A^3-8A^2-10A-16I = 0$

(D) $A^3-10A^2+8A-16I = 0$

20. The value of $x P'_{n+1} - P'_{n-1}$ is

(A) $n P_n(x)$

(B) n^2

(C) $P'_{n-1}(x)$

(D) None of these

21. The value of $P'_{n+1}(x) - P'_{n-1}(x)$ is

(A) $P_n(x)$

(B) $(2n+1)\,P_n(x)$

(C) $P'_n(x)$

(D) None of these

22. The value of $P_n(1)$ is

(A) 0

(B) 1

(C) -1

(D) None of these

23. If $P_n(x)$ is a solution of Legendre's equation, then value of $P_0(x)$ will be

(A) 1

(B) 0

(C) -1

(D) x

24. $nJ_n(x) - x\,J_{n+1}(x) = ?$

(A) $x\,J_n(x)$

(B) $(n+1)\,J'_n(x)$

(C) $x\,J'_n(x)$

(D) None of these

25. $-n\,J_n(x) + x\,J_{n-1}(x) = ?$

(A) $J'_{n-1}(x)$

(B) $x\,J'_n(x)$

(C) $J'_{(n-1)}(x)$

(D) None of these

26. $x[J_{n-1}(x) + J_{n+1}(x)] = ?$

(A) $2J_n(x)$

(B) $2\,J'_n(x)$

(C) $2n\,J_n(x)$

(D) None of these

27. The value of

$\frac{1}{2^n.n!}\frac{d^n}{dx^n}(x^2-1)^n$ is

(A) $P_n'(x)$

(B) $Q_n(x)$

(C) $P_{n-1}(x)$

(D) $P_n(x)$

28. The value of $\oint_C \frac{1}{z^2+4}dz$ where C is $|z-2i|=1$ will be

(A) 0

(B) $\frac{1}{5}$

(C) $\pi/2$

(D) $\pi/3$

29. The value of $\oint_C \frac{dz}{z^2+6}$, where C is the boundary of $|z-i| = 1$ is

(A) $2\pi i$

(B) $4\pi i$

(C) 0

(D) πi

30. Evaluate $\oint_C \frac{1}{(z-1)^3(z-3)}dz$ where C is the rectangular region defined by $x=0$, $x=4$, $y=-1$ and $y=1$

(A) 1

(B) 0

(C) $\frac{\pi}{2}i$

(D) $\pi(3+2i)$

31. The value of $\int_0^{\pi} \sin^2\theta.\cos^4\theta.d\theta$ is

(A) π

(B) 2π

(C) $\pi^2/32$

(D) $\pi/16$

32. The residue at the singular point $z = -2$ of $f(z) = \frac{1+z+z^2}{(z-1)^2(z+2)}$

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) 4/3

(D) 3/2

33. Residue at $z=2$ of $f(z)=\frac{2z+1}{z^2-z-2}$ is

(A) 5/3

(B) 1/3

(C) 3/5

(D) 2/3

34. $f(z) = u(x,y) + iv(x,y)$ is an analytic function of complex variable $z=x+iy$ then $u(x,y)$ equals

(A) x^2+y^2

(B) x^2-y^2

(C) $\frac{1}{2}(x^2+y^2)$

(D) $\frac{1}{2}(x^2-y^2)$

35. Which of the following function $f(z)$, of the complex variable z, is not analyic at all the points of the complex plane?

(A) $f(z) = z^2$

(B) $f(z) = e^z$

(C) $f(z) = \sin z$

(D) $f(z) = \log z$

36. Find the value of

$$\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}\, d\theta$$

(A) π

(B) $\pi/3$

(C) $\frac{2\pi}{3}$

(D) $\pi/6$

37. Residues at two poles of the function $f(z)=\frac{z}{z^2+4}$ are

(A) Equal and opposite in sign

(B) Equal with same sign

(C) All together different

(D) Inverse of each other

38. What is the sum of residues of function $f(z)=\frac{e^z}{z^2+a^2}$ at all its poles?

(A) $\frac{\sin a}{a}$

(B) $-\frac{\sin a}{a}$

(C) $\frac{\cos a}{a}$

(D) $-\frac{\cos a}{a}$

39. What is the value of integral $\int f(z)\,dz$ around a circle of radius z with its centre at the origin if $f(z) = \frac{1}{(z-1)}$?

(A) Zero

(B) πi

(C) $4\pi i$

(D) $2\pi i$

40. The eigen values of the matrix

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$ are

(A) 1, 1, 2

(B) 1, 2, 3

(C) 2, 2, 3

(D) 1, 1, 3

41. $\vec{F} = \hat{i}(2xy + z^3) + \hat{j}(x^2) + \hat{k}(3xz^2)$ is a

(A) Solenoidal vector

(B) Conservative force field

(C) Non conservative force field

(D) None of the above

42. Rank of Matrix $A = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{vmatrix}$ is

(A) 0

(B) 1

(C) 3

(D) 2

43. The directional derivative of $\phi = xyz$ at the point (1,1,1) in the direction $\hat{i}$ is :

(A) -1

(B) -1/3

(C) 1

(D) 1/3

44. If a matrix A can be written as $A^2 = A$ then A is called

(A) Nilpotent Matrix

(B) Involuntary Matrix

(C) Singular Matrix

(D) Idempotent Matrix

45. Which of the following is a Hermitian Matrix :

(A) $\begin{vmatrix} i & 1+i \\ -1+i & -2i \end{vmatrix}$

(B) $\begin{vmatrix} 4 & 3-2i \\ 3+2i & -4 \end{vmatrix}$

(C) $\begin{vmatrix} i & -2+3i \\ 2+3i & 0 \end{vmatrix}$

(D) $\begin{vmatrix} -i & 1 & 2+i \\ -1 & 0 & 3i \\ -2+i & 3i & i \end{vmatrix}$

46. The value of $J_0(0)$ is

(A) 0

(B) -1

(C) 1

(D) None of these

47. Bessel's Differential Equation is

(A) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(x+1)y = 0$

(B) $x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + ny = 0$

(C) $x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} + (x^2 - n^2)y = 0$

(D) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-n^2)y = 0$

48. The value of $H_2(x)$ is

(A) 1

(B) 2x

(C) $4x^2-2$

(D) $8x^3-12x$

49. The value of $J_{\frac{1}{2}}(x)$ is equal to

(A) $\sqrt{\frac{2}{\pi x}}\sin x$

(B) $\sqrt{\frac{\pi x}{2}}\sin x$

(C) $\sqrt{\frac{2}{\pi x}}\cos x$

(D) $\sqrt{\frac{\pi x}{2}}\cos x$

50. If $u = x^2-y^2$, then the Conjugate harmonic function is

(A) 2xy

(B) x^2+y^2

(C) y^2-x^2

(D) $-x^2-y^2$

51. Function z^2 is

(A) Analytic everywhere

(B) Analytic at origin only

(C) Not analytic everywhere

(D) Analytic in the upper half plane only

52. Which of the following functions is analytic everywhere?

(A) $|z|$

(B) Re Z

(C) z^{1}

(D) sin z

53. What is the modulus of $\left(\frac{2+i}{3-i}\right)^2$?

(A) 2/3

(B) 2

(C) 3

(D) $\sqrt{2}$

54. What is the imaginary part of $\log_e z$?

(A) log y/x

(B) $\tan^{-1} y/x$

(C) e^{-yx}

(D) None of these

55. Which of the following is not correct?

(A) $H_{2n}(0) = (-1)^n (2n)! / n!$

(B) $H_{(2n+1)}0 = 0$

(C) $H'_{2n}0 = 0$

(D) $H'_{(2n+1)}(0) = 0$

56. Given $\frac{d^m}{dx^m} H_n(x) = A\ H_{n-m}(x)$, Find A

(A) $2^n \underline{|n}$

(B) $2^m \underline{|n}$

(C) $2^m \frac{\underline{|n}}{\underline{|n-m}}$

(D) $2^n \frac{\underline{|n}}{\underline{|n-m}}$

57. What is the value of $J_{-1/2}(\pi/2)$?

(A) Zero

(B) One

(C) $\pi/2$

(D) None of these

58. If n is a positive integer, what is the value of the following integral

$$\int_{-1}^{1} P_n(x)(1-2xh+h^2)^{-1/2}\,dx$$

(A) $\frac{2}{2n+1}$

(B) Zero

(C) $\frac{2h^n}{2n+1}$

(D) $\frac{2h^n}{2n+1}$

59. In the general solution of second order differential Equation $\frac{d^2y}{dx^2} - 2\alpha\frac{dy}{dx} + \alpha^2 y = 0$ one term constant $e^{\alpha x}$, then its second term will be a constant times

(A) $e^{-\alpha x}$

(B) $xe^{-\alpha x}$

(C) $xe^{\alpha x}$

(D) $\frac{1}{x}e^{\alpha x}$

60. If one of the eigen value of the matrix $\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix}$ is zero, what is the other eigen value?

(A) 1

(B) -1

(C) 2

(D) -2

61. For two vectors $A = A_x i + A_y j + A_z k$ and $B = B_x i + B_y j + B_z k$ is

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$$

then the given vecters are necessarily

(A) equal

(B) parallel

(C) perpendicular

(D) collinear

62. The time derivative of unit vecter i_θ of cylindrical coordinates is

(A) Zero

(B) $\dot{\theta} i_\theta$

(C) $\dot{r} i_\theta$

(D) $-\dot{\theta} i_r$

63. The component of acceleration along θ-coordinate of spherical coordinate system is

(A) $r\ddot{\theta}$

(B) $r\ddot{\theta} + 2\dot{r}\dot{\theta}$

(C) $r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\sin\theta\cos\phi\dot{\phi}^2$

(D) $r\ddot{\theta} - r\sin\theta\cos\phi\,\dot{\phi}^2$

64. Two equal drops of water are falling through air with the steady velocity v. If the dropes Collapses, what will be the new velocity?

(A) $2v$

(B) $\frac{v}{2}$

(C) $2^{1/3}v$

(D) $2^{2/3}v$

65. What is the ratio $\frac{\Gamma(-3/2)}{\Gamma(3/2)}$?

(A) 1

(B) $\frac{3}{8}$

(C) $\frac{8}{3}$

(D) $\frac{2}{3}$

66. Given that

$$\int_0^1 \frac{dx}{(1-x^n)^{1/n}} = P\Gamma(P)\,\Gamma(1-P)$$

Find P

(A) n

(B) $\frac{1}{n}$

(C) (n−1)

(D) $\frac{1}{n-1}$

67. Product of two orthogonal matrices is

(A) Symmetric

(B) Skew-symmetric

(C) Orthogonal

(D) None of these

68. If A is real anti-symmetric matrix, then A is orthogonal when and only when :

(A) $A^2=1$

(B) $A^2=0$

(C) $A^2=-1$

(D) $A=0$

69. If inverse of $\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix}$ is $\begin{vmatrix} 0 & 2a & -a \\ -3a & a & a \\ 3a & -2a & a \end{vmatrix}$, Find a.

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{\sqrt{2}}$

70. The matrix $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ is

(A) Only Hermition

(B) Only Unitary

(C) Hermition and Unitary

(D) Neither Hermition nor unitary

71. What is the nature of purely imaginary Hermition matrix with vanishing elements along its principal diagonal?

(A) Symmetric

(B) Anti Symmetric

(C) Orthogonal

(D) None of these

72. θ if AX= λX, AY=μy and $\lambda \neq \mu$ what is (A-λI)Y ?

(A) 0

(B) μY

(C) $(\mu - \lambda)$ y

(D) $(1 - \lambda)$ y

73. Find the eigen values of

$$\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

(A) $\pm e^{i\theta}$,

(B) $e^{\pm i\theta}$

(C) $\cos\theta \pm \sin\theta$

(D) None of these

74. Every Harmonic function satisfies

(A) Bessel's function

(B) Poisson's equation

(C) Laplace equation

(D) All of the above

75. All the diagonal elements of a skew Hermition Matrix are :

(A) Real only

(B) Zero only

(C) imaginary only

(D) Either zeros or pure imaginary